Various Issues in the Large Strain Theory of Trusses

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1 Manifold View of a Truss

Within the large strains regime, manifold theory is used in order to define a truss in the most fundamental way. The truss is defined as a three–dimensional differentiable manifold, whose particles are identified via three parameters (s_1, s_2, s_3) that belong to a closed set of the set R of real numbers, with the controlling one–dimensional submanifold s_1 being globally homeomorphic to R. We postulate the existence of a one parameter family $A(s_1) = A$ of bounded areas in R^2 that describes the cross section of the prismatic truss.

All possible truss configurations that are generated–by–embeddings of the truss manifold in the Euclidean space are denoted by ${}^{m}C$, with m corresponding to some possible configuration labeling, such that $m = 0$ gives the natural reference configuration [2].

2 Truss Kinematics

Acceptable homogeneous embeddings of the s_1 truss submanifold in the Euclidean space are assumed to correspond to straight line segments having length ${}^{m}L$ (determined solely from the global coordinates of its ends), while acceptable homogeneous embeddings of the s_2–s_3 truss submanifold correspond to bounded plane surfaces having area ${}^{m}A$ and being normal to the embedded s_1 line.

Unique orthogonal frames can be constructed in each configuration, and, assuming mutually orthogonal cross–section spanning vectors, it is shown that for the truss, Hill's large strain measures (which measure the strain experienced by the truss in going from some arbitrary reference configuration ${}^{n}C$ to ${}^{m}C$) are given by [1, 4, and 5]:

$$ {}_{n}^{m}\boldsymbol{\varepsilon}^{(k)} = \frac{1}{k}\left({}_{n}^{m}\boldsymbol{\Lambda}^{k} - \mathbf{I}\right), \qquad k = \ldots\text{-}2, \text{-}1, 0, 1, 2, \ldots \tag{1} $$

where ${}_{n}^{m}\boldsymbol{\Lambda}$ is a diagonal stretch matrix [1–3], and $\mathbf{I}$ is the unit matrix.

3 Stress – Strain Conjugacy

Following [4, 5], it is found that conjugacy between various stress and strain measures, depends on the reckoned truss's volume. Some conjugacy issues were studied in the test case that follows.

Let's assume that the truss's strain energy vanishes at the natural reference configuration. Using the simplest, linear hyperelastic constitutive law and the truss's boundary conditions (vanishing stresses at the truss's bounding surface except at the end cross section), while reckoning conjugacy with respect to the current truss volume ${}^{m}V$, it is found that the truss's strain energy ${}^{m}U$, is:

$$ {}^{m}U = {}^{m}L\, {}^{m}A \frac{{}_{0}E^{(k)}_{{}^{m}V}}{2} \left({}^{m}_{0}\boldsymbol{\varepsilon}^{(k)} \right)^{2}_{11} + {}^{0}L\, {}^{0}N \left({}^{m}_{0}\boldsymbol{\varepsilon}^{(k)} \right)_{11} \tag{2} $$

where, $\left({}^{m}_{0}\boldsymbol{\varepsilon}^{(k)} \right)_{11}$ is the first component of the ${}^{m}_{0}\boldsymbol{\varepsilon}^{(k)}$ tensor. At the same time, it is found, that the cross sectional area of the truss should satisfy:

$$ {}^{m}A = {}^{0}A \left[1 + {}_{0}n^{(k)}_{{}^{m}V} - {}_{0}n^{(k)}_{{}^{m}V}\, {}^{m}L^{k}\, {}^{0}L^{-k} \right]^{2/k} \tag{3} $$

${}^{0}N$ is the prestressing force, whereas ${}_{n}E^{(k)}_{{}^{m}V}$ and ${}_{n}n^{(k)}_{{}^{m}V}$ are constitutive fourth order tensors that depend on the adopted strain measure (superscript), on the reference configuration chosen (left subscript), as well as on the volume used to define conjugacy (right subscript). A particular form of (2) is found in [1] for $k = 0$ (Logarithmic Strain).

If instead of the current truss volume ${}^{m}V$, the natural reference volume ${}^{0}V$ is used in defining conjugacy, we find (keeping the same type of material law) that the truss's strain energy is:

$$ {}^{m}U = {}^{0}L\, {}^{0}A \frac{{}_{0}E^{(k)}_{{}^{0}V}}{2} \left({}^{m}_{0}\boldsymbol{\varepsilon}^{(k)} \right)^{2}_{11} + {}^{0}L\, {}^{0}N \left({}^{m}_{0}\boldsymbol{\varepsilon}^{(k)} \right)_{11} \tag{4} $$

By (2), (3) and (4), we see that when conjugacy is reckoned with respect to ${}^{0}V$, no constraint of the type (3) is needed, whereas when conjugacy is defined with respect to ${}^{m}V$, (3) is needed in order to comply with this simple hyperelastic constitutive law and the truss's boundary conditions.

4 Concluding Remarks

Results of the present work indicate that if conjugacy is reckoned with respect to the current volume, then even in this simple test case, the structural behavior of the truss depends on its cross sectional kinematics. Hence, when the strains are large, the way the truss's cross section changes shape and area, affects its structural behavior. Thus, the large strains truss is like a semi–one–dimensional problem, whose proper definition requires fundamental understanding of its kinematics, kinetics, constraints, constitutive laws and conjugacy. In other words, quoting [3], "…with little exaggeration, *there are no one - dimensional problems*…".

References

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